AC Magnetic Susceptibility Analytical Study of (Bi, Pb)Sr-Ca-Cu-O Superconducting System

Using Bean Critical State Model

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Abstract

AC magnetic susceptibility $\chi_{ac}$ as a function of temperature $T$ and applied field amplitude $H$ of (Bi$_{1.6}$Pb$_{0.4}$)Sr$_2$Ca$_2$Cu$_2$O$_8$ and (Bi$_{1.3}$Pb$_{0.7}$)Sr$_2$Ca$_2$Cu$_2$O$_8$ superconducting system were analytical studied using Bean critical state model. The data was taken from the ac susceptibility experiment, critical temperature and grain volume fraction of specimens were utilized from the real part of ac susceptibility measurement while the pinning force density was utilized from the imaginary part of the susceptibility measurement.

The calculation from Mathematica program indicated that (Bi$_{1.6}$Pb$_{0.4}$)Sr$_2$Ca$_2$Cu$_2$O$_8$ superconductor of nominal composition (2234), prepared by solid reaction with an intermediate ground and pressed, provided biggest hysteresis loss, and the calculated $\chi_{ac}(H,T)$ results were comparable with the experimental $\chi_{ac}(H,T)$ data.

Key Words: BPSCCO, $\chi_{ac}$, critical state, Bean model

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Introduction

The critical states of sintered high temperature superconductors, which contain weakly coupled superconductive grains have been studied by ac magnetic susceptibility, $\chi = \chi' + i\chi''$, measurements (Chen et al., 1990; Ishida and Goldfarb, 1990; Celebi et al., 1998). Measurement of the superconducting transition by means of complex ac susceptibility provides a sharp decreased in $\chi'$ due to diamagnetic shielding and a peak in $\chi''$ representing losses. Both $\chi'$ and $\chi''$ are sensitive to both temperature $T$ and the amplitude of the ac magnetic field $H$ (Trivijitkasem and Sratongluan, 1999).

The ceramic superconductors composed of an array of superconducting grains which interconnected by weak links. Consequently, the intergranular and granular pinning – depinning properties were used to define irreversibility line in the H-T plane (Gonzalez et al., 1995). Below this line, a finite critical current density exists, while above this line, it turns to zero. Some critical state models are employed to calculate $\chi_{ac}$ of ceramic superconductor, such as Rollins-Silcox (R-S), Kimishima, Bean, and Anderson-Kim model. Ji et al., (1989) proposed a macroscopic critical state model to predict both odd and even harmonic susceptibility in high-Tc superconductors. Lee and Kao (1995) used Anderson-Kim critical state model to analyze $\chi_{ac}$ of YBCO ceramic superconductors which showed satisfactory agreement with the experimental results.

In the present work, Bean critical state model was used for analyzing the fundamental susceptibility $\chi_{ac}$ as a function of temperature $T$ and ac magnetic field amplitude $H$ of high Tc BSCCO superconductors. The calculated $\chi_{ac}(H,T)$ results were compared with the measuremental data on thin slab sample immersed in a pure ac field.

Theory

When a superconductor is put in an ac magnetic field $H = H_{ac}\cos(\omega t)$, the Fourier expansion of time dependent magnetization $M(t)$ is given by

$$M(t) = H_{ac}\sum_{n=1}^{\infty}[\chi'_{n}\cos(n\omega t) + \chi''_{n}\sin(n\omega t)],$$

(1)

where $n = 1, 2, 3, \ldots$, $\chi'_{n}$ and $\chi''_{n}$ are the n-th order component of real and imaginary part of complex ac susceptibility, respectively.

The real and imaginary fundamental susceptibility $\chi'_{0}$ and $\chi''_{0}$, $(n$ equals to $1$), can be calculated from the following expressions

$$\chi'_{0} = \frac{1}{\pi} \int_{0}^{\pi} m(t)\cos(\omega t)\,d(\omega t),$$
\[ \chi'_0 = \frac{1}{\pi} \int_0^{2\pi} m(t) \sin(\omega t) \, d(\omega t), \]  

where reduced magnetization \( m(t) = \frac{M(t)}{H} \).

Many different theoretical models have been used to describe the hysteretic behavior of superconductor, from which the critical current density can be analyzed. Among them, the Bean model is the simplest and appropriate one for analyzing the hysteretic effect. The critical state model proposed by Bean assumed that the critical current density \( J_c \) is constant at the small total field \( H \). Then the pinning force density, i.e. Lorentz force per volume, \( \alpha = J_c B \) holds for the equilibrium state of the local flux density \( B \) in the sample.

In order to calculate hysteresis loop of reduced magnetization \( m(t) \) and reduced ac field \( h(t) = H/H_\infty = \cos(\omega t) \), the virgin magnetization \( m_{\text{vir}} = \langle B \rangle / \langle \mu_0 \rangle - h \), is introduced, where \( \langle B \rangle \) is the average local field for the inner space of the sample.

For the slab sample, the virgin magnetization \( m_{\text{vir}} \) can be expressed by

\[ m_{\text{vir}} = -\frac{\alpha}{2} + \frac{(h-\alpha)^2}{2\alpha}, \quad \text{for } 0 \leq h \leq \alpha, \]
\[ = -\frac{\alpha}{2}, \quad \text{for } \alpha \leq h \leq 1. \]

For \( 0 \leq \alpha \leq 1 \), the time dependent reduced magnetization \( m(t) \) are given by

\[ m(t) = \frac{\alpha}{2} - \frac{1}{4\alpha} \{\cos(\omega t) - \cos \theta \}^2, \quad \text{for } 0 \leq \omega t \leq \theta, \]
\[ = \frac{\alpha}{2}, \quad \text{for } \theta \leq \omega t \leq \pi, \]
\[ = -\frac{\alpha}{2} + \frac{1}{4\alpha} \{\cos(\omega t) + \cos \theta \}^2, \quad \text{for } \pi \leq \omega t \leq \pi + \theta, \]
\[ = -\frac{\alpha}{2}, \quad \text{for } \pi + \theta \leq \omega t \leq 2\pi. \]

where \( \theta = \cos^{-1}(1-2\alpha) \).

For \( 1 \leq \alpha \), the reduced magnetization \( m(t) \) became as the followings:

\[ m(t) = \frac{1}{2\alpha} \cos \theta \cos(\omega t) - \frac{1}{4\alpha} \{\cos^2(\omega t) - 1\}, \quad \text{for } 0 \leq \omega t \leq \pi, \]
\[ = \frac{1}{2\alpha} \cos \theta \cos(\omega t) + \frac{1}{4\alpha} \{\cos^2(\omega t) - 1\}, \quad \text{for } \pi \leq \omega t \leq 2\pi. \]

In order to calculate \( \chi'_{\text{g}}(H, T) \), the following expressions are introduced:

\[ \chi'_0 = (1-f_\text{g})\chi'_{\text{sm}} + f_\text{g}\chi'_{\text{ot}}, \]
\[ \chi''_0 = (1-f_\text{g})\chi''_{\text{sm}} + f_\text{g}\chi''_{\text{ot}}, \]  

where \( f_\text{g} \) is the effective volume fraction of superconducting grain, \( \chi'_{\text{sm}} \) and \( \chi''_{\text{sm}} \) are the components of grain susceptibility.
The theoretical matrix susceptibilities $\chi'_{0m}$, $\chi''_{0m}$ and $\chi'_{0g}$, $\chi''_{0g}$ are derived by performing the following analytically integrations, (only the expressions for intergranular susceptibilities are shown here):

$$\chi'_{0m} = \frac{1}{\pi a_m} (1-2a_m) \sin^{-1} \sqrt{\frac{a_m}{(1-4a_m(1-a_m))}} \sqrt{\frac{1-a_m}{1-a_m}}, \quad \text{for } 0 \leq a_m \leq 1,$$

$$\chi''_{0m} = \frac{2}{3\pi} a_m (3-2a_m), \quad \text{for } 0 \leq a_m \leq 1,$$

$$\chi'_{0m} = \frac{1}{2a_m} - 1, \quad \text{for } 1 \leq a_m,$$

and

$$\chi''_{0m} = \frac{2}{3\pi a_m}, \quad \text{for } 1 \leq a_m. \quad (5)$$

The pinning force density $\alpha$ as a function of temperature consisted of intergranular matrix pinning force density $\alpha_m(T)$ and granular pinning force density $\alpha_g(T)$. For (B, P)SCCO superconducting system, the pinning force densities are assumed as the followings:

$$\alpha_m(T) = \alpha_m(0) \left( \frac{T}{T_{cm}} \right)^{1.9}, \quad (6)$$

$$\alpha_g(T) = \alpha_g(0) \left( \frac{T}{T_{cg}} \right)^{2}, \quad (7)$$

where $\alpha_g(0)$ is the granular pinning force density at 0 K, and $T_{cm}$, $T_{cg}$ are Josephson intergranular, granular critical temperature, respectively.

According to Bean model, $\alpha(T)$, at $\chi''_{\text{omax}}$, equals to 0.75, then $\alpha(0)$ can be determined from the following equations:

$$\alpha_m(0) = 0.75 \sqrt{\left[ 1 - \left( \frac{T_p}{T_{cm}} \right) \right]^{1.9}}, \quad (8)$$

and

$$\alpha_g(0) = 0.75 \sqrt{\left[ 1 - \left( \frac{T_p}{T_{cg}} \right) \right]^{2}}, \quad (9)$$

where $T_p$ is the temperature at $\chi''$ (maximum).

Material and Methods

(Bi_{1.6}Pb_{0.4})Sr_2Ca_2Cu_3O_{β} and (Bi_{1.7}Pb_{0.3})Sr_2Ca_2Cu_3O_{γ} superconducting system were prepared by the conventional solid state reaction technique with an intermediate ground and pressed (P) or none (N). Critical temperature $T_c$, granular volume fraction $f_g$ were taken from our previous study (Sirinilakul, et al., 1998). Pinning force density of grain $\alpha_g$ and intergrain $\alpha_m$ at absolute temperature of the samples were determined from experimental ac susceptibility measurement data performed by zero field cooling (ZFC) method.
Results and Discussion

The magnetization curves $m(h)$ derived from Bean model, equation (3), are presented in Fig. 1. These curves show a hysteretic behavior that the biggest hysteretic loss belongs to the $(\text{Bi}_{1.6}\text{Pb}_{0.4})\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_\beta$ (P) superconductor prepared by an intermediate ground and pressed, the sample prepared without an intermediate ground and pressed provides smaller hysteretic loss. All the samples showed parallelepipeds shape of hysteretic behavior, which were consistent with the results reported by Camacho et al. (1997), and Gotoh et al. (1990).

Fig. 1 Calculated hysteresis loops of the thin slab superconductors at 101 K and field amplitude 0.1 A/m.

Fig. 2 The observed ac susceptibility $\chi$ at 125 Hz and field amplitude, from right to left, $H = 0.1, 1, 10, 100, 200, 300, 400$ and 500 A/m, respectively.
Table 1 Critical temperature $T_c$, granular volume fraction $f_s$, pinning force density of grain $a_s(0)$ and intergrain $a_m(0)$ at 0 K of (Bi, Pb)Sr-Ca-Cu-O superconductors.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$T_c$ (K)</th>
<th>$f_s$</th>
<th>$a_s(0)$</th>
<th>$a_m(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>($\text{Bi}<em>{1.6}\text{Pb}</em>{0.4})\text{Sr}_2\text{Ca}_2\text{Cu}<em>3\text{O}</em>\beta$ (N)</td>
<td>106.2</td>
<td>0.26</td>
<td>616.6</td>
<td>146.7</td>
</tr>
<tr>
<td>($\text{Bi}<em>{1.6}\text{Pb}</em>{0.4})\text{Sr}_2\text{Ca}_2\text{Cu}<em>3\text{O}</em>\beta$ (P)</td>
<td>105.0</td>
<td>0.28</td>
<td>1285.8</td>
<td>148.1</td>
</tr>
<tr>
<td>($\text{Bi}<em>{1.7}\text{Pb}</em>{0.3})\text{Sr}_2\text{Ca}_2\text{Cu}<em>3\text{O}</em>\gamma$ (N)</td>
<td>107.0</td>
<td>0.18</td>
<td>557.0</td>
<td>377.7</td>
</tr>
<tr>
<td>($\text{Bi}<em>{1.7}\text{Pb}</em>{0.3})\text{Sr}_2\text{Ca}_2\text{Cu}<em>3\text{O}</em>\gamma$ (P)</td>
<td>105.2</td>
<td>0.26</td>
<td>2166.9</td>
<td>182.7</td>
</tr>
</tbody>
</table>

Fig. 3 The calculated ac susceptibility $\chi$ at 125 Hz and field amplitude, from right to left, $H = 0.1, 1, 10, 100, 200, 300, 400$ and $500$ A/m, respectively.

The measured $\chi'$ and $\chi''$ as a function of temperature $T$ and field amplitude $H = 0.1, 1.0, 10, 100, 200, 300, 400$ and $500$ A/m of ($\text{Bi}_{1.6}\text{Pb}_{0.4})\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_\beta$ and ($\text{Bi}_{1.7}\text{Pb}_{0.3})\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_\gamma$ superconductors are shown in Fig. 2. Table 1 listed the data deduced from the measured $\chi$ data. The calculated $\chi'(H, T)$ and $\chi''(H, T)$ at the same field amplitude are also shown in Fig. 3.

A comparison of the observed $\chi_n(H, T)$ behaviors with the calculated results, at $H = 0.1$ A/m, of the superconducting system are presented in Fig. 4, which can be seen that the superconducting system prepared by an intermediate ground and pressed provides better consistent of the calculated and observed ac susceptibility $\chi_{ac}(H, T)$. This is probably due to the smaller grain size and more dense of the micro-structure of the intermediate ground and pressed sample.
(Trivijtkasem and Sratongluan, 2000) which leads to the better-linked between superconduction grain; and the pinning force densities $a_m(0)$ and $a_g(0)$ determined from equation (8) and (9) using the experimental data are more appropriate for calculating magnetization curves derived from Bean critical state model. Hence, Bean critical state model can be used for reproducing the observed $\chi$ data, especially at lower H-field of the (Bi$_{1.6}$Pb$_{0.4}$) and (Bi$_{1.7}$Pb$_{0.3}$) superconductors prepared by an intermediate ground and pressed.

**Conclusion**

Bean critical state model was employed for analyzing the fundamental ac susceptibility as a function of temperature $T$ and ac magnetic field amplitude $H$ of high Tc (Bi$_{1.6}$Pb$_{0.4}$)Sr$_2$Ca$_2$Cu$_3$O$_\gamma$ and (Bi$_{1.7}$Pb$_{0.3}$)Sr$_2$Ca$_2$Cu$_3$O$_\gamma$ superconducting system. Pinning force density was determined from the susceptibility measurement data performed by zero field cooled method. The calculation from Mathematica program indicated that the biggest hysteresis loss belonged to the (Bi$_{1.6}$Pb$_{0.4}$) superconductor of nominal composition (2234) prepared by solid reaction with an intermediate ground and pressed. Bean critical state model was used to reproduce the observed $\chi$ data. The results showed a better consistent of the calculated and observed ac susceptibility $\chi$, especially at 0.1 A/m field amplitude of the (Bi, Pb) superconducting system prepared by an intermediate ground and pressed.

![Graphs of ac susceptibility vs temperature](image)

**Fig. 4** A comparison of the observed and calculated $\chi(H,T)$ at 125 Hz and $H = 0.1$ A/m.


Sirinilakul, S., S. Trivijitkasem and N. Prungleg. 1998. First harmonic ac susceptibility of high Tc (Bi$_x$Pb$_{1-x}$)$_2$Sr$_2$Ca$_2$Cu$_3$O$_{10-}$ superconductor system, (x = 0, 0.1, 0.2, ... 0.5). Proc. of the 36th Kasetsart U. conf. 1-7.
